



The Thomas Jefferson National Accelerator Facility
Theory Group Preprint Series

Additional copies are available from the authors.

The Southeastern Universities Research Association (SURA) operates the Thomas Jefferson National Accelerator Facility for the United States Department of Energy under contract DE-AC05-84ER40150.

The Goldberger-Treiman Discrepancy in $SU(3)$

José L. Goity^{1,2}, Randy Lewis^{1,3}, Martin Schvellinger^{1,2,4} and Longzhe Zhang^{1,2}

¹*Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA, 23606, U.S.A.*

²*Department of Physics, Hampton University, Hampton, VA, 23668, U.S.A.*

³*Department of Physics, University of Regina, Regina, SK, S4S 0A2, Canada*

⁴*Department of Physics, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina*

Abstract

The Goldberger-Treiman discrepancy in $SU(3)$ is analyzed in the framework of heavy baryon chiral perturbation theory (HBChPT). It is shown that the discrepancy at leading order is entirely given by counterterms from the $\mathcal{O}(p^3)$ Lagrangian, and that the first subleading corrections are suppressed by two powers in the HBChPT expansion. These subleading corrections include meson-loop contributions as well as counterterms from the $\mathcal{O}(p^5)$ Lagrangian. Some one-loop contributions are calculated and found to be small. Using the three discrepancies (πNN , $KN\Lambda$ and $KN\Sigma$) which can be extracted from

I. INTRODUCTION

The Goldberger-Treiman relation (GTR) [1], obtained from matrix elements of the divergence of axial currents between spin 1/2 baryons, is an important indicator of explicit chiral symmetry breaking by the quark masses. It interrelates baryon masses, axial vector couplings, the baryon-pseudoscalar meson (Goldstone boson \equiv GB) couplings and the GB decay constants. Explicit chiral symmetry breaking leads to a departure from the GTR (defined below) which is called the Goldberger-Treiman discrepancy (GTD).

The GTD has been repeatedly discussed over time [2] and for several reasons there were difficulties in arriving at a clear understanding. On one hand, there was no available effective theory with a systematic expansion to address the problem, and on the other hand the experimental values of the baryon-GB couplings were too poorly known. In recent years, progress has been made on both fronts. There is now a baryon chiral effective theory that permits a consistent expansion of the discrepancy [3–6]. There has also been progress in the determinations of the baryon-GB couplings that are the main source of uncertainty in the phenomenological extraction of the discrepancies. In fact, the current knowledge of the couplings $g_{\pi NN}$, $g_{KN\Lambda}$ and, to a lesser extent, $g_{KN\Sigma}$ is good enough to justify a new look at the GTD in SU(3). In this work we study the GTD in the light of heavy baryon chiral perturbation theory (HBChPT) [4,5].

Let us first briefly review the derivation of the GTR [7] and the definition of the GTD.

where M_a is a baryon mass. Crucial to the derivation of the GTR is the GB pole contribution represented in Fig. 1. To explicitly expose the pole term, the matrix element in Eq. (2) can be rewritten as

$$\langle b, p_b | \partial^\mu A_\mu^c | a, p_a \rangle = i\bar{U}_b(p_b) \frac{N^{abc}(q^2)}{q^2 - m_c^2 + i\epsilon} \gamma_5 U_a(p_a) , \quad (3)$$

where $N^{abc}(q^2) = g_{cab}(q^2)P^c(q^2) + (q^2 - m_c^2)\delta^{abc}(q^2)$, m_c is a GB mass, and $g_{cab}(q^2)$ the baryon-GB form factor, defined such that in the physical basis of the Gell-Mann matrices $g_{3,6+i7,6-i7}(M_\pi^2)$ is equal to $g_{\pi^0 nn}$, etc. $P^c(q^2)$ represent the couplings of the pseudoscalar currents to the GB's, given in the chiral limit by $P^c = m_c^2 F_c$ (F_c is the decay constant, where $F_\pi = 92.42$ MeV); the q^2 dependence of $P^c(q^2)$ starts at $\mathcal{O}(p^4)$ and is henceforth disregarded. Finally, $\delta^{abc}(q^2)$ denotes contributions not involving the GB pole, and it starts as a quantity of $\mathcal{O}(p^2)$. This separation of pole and non-pole contributions is not unique (the off-shell functions separately are not observables); for instance, up to higher order terms in q^2 , we can choose to remove the q^2 dependence in $g_{cab}(q^2)$ around the point $q^2 = m_c^2$ by a simple redefinition of δ^{abc} .

In the chiral limit $\partial^\mu A_\mu^c = 0$, and at $q^2 = 0$ Eq. (2) gives:

$$M g_A^{abc}(0) = \lim_{q^2 \rightarrow 0} q^2 g_2^{abc}(q^2) = F_c g_{cab}(0) , \quad (4)$$

which is the general form of the GTR. Here M is the common octet baryon mass in the chiral limit. In the real world, chiral symmetry is explicitly broken by the quark masses and the GB's become massive. In this case, Eqs. (2) and (3) lead to

Notice that while the GTR, Eq. (4), is defined at $q^2 = 0$, the GTD in Eq. (7) is given at $q^2 = m_c^2$ because only at that point is the coupling g_{cab} unambiguously determined. At leading order in the quark masses, the GTD can then be expressed as follows:

$$\Delta^{abc} = m_c^2 \frac{\partial}{\partial q^2} \log N^{abc}(q^2) |_{q^2=m_c^2} . \quad (8)$$

II. TREE LEVEL CONTRIBUTIONS

Throughout we are going to use standard definitions, namely:

$$u \equiv \exp \left(-i \frac{\pi^a \lambda^a}{2F_0} \right) , \quad (9)$$

$$\chi \equiv 2B_0 (s + i p) , \quad (10)$$

$$\chi_{\pm} \equiv u^\dagger \chi u^\dagger \pm u \chi^\dagger u , \quad (11)$$

$$\omega^\mu \equiv \frac{i}{2} (u^\dagger \partial^\mu u - u \partial^\mu u^\dagger) , \quad (12)$$

$$S_\nu^\mu \equiv \frac{i}{2} \gamma_5 \sigma^{\mu\nu} v_\nu . \quad (13)$$

The HBChPT Lagrangian is ordered in powers of momenta and GB masses, which are small compared to both the chiral scale and the baryon masses,

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \dots . \quad (14)$$

Although the Lagrangian is written as a single expansion, it will be useful to keep track of the chiral and $1/M$ suppression factors separately. As will be demonstrated explicitly below,

are two types of terms which contribute to the GTD. The first type must contain the pseudoscalar source χ_- . The second type must contain monomials such as $[\mathcal{D}^\mu, [\mathcal{D}_\mu, \omega_\nu]]$ and $[\mathcal{D}^\nu, [\mathcal{D}_\mu, \omega^\mu]]$ between the baryon field operators (here \mathcal{D}^μ is the chiral covariant derivative). However, upon using the classical equations of motion satisfied by the GB fields at $\mathcal{O}(p^2)$, it turns out that terms of the second type can be recast into terms among which there are terms of the first type. In this way, one moves the explicit q^2 dependence from g_{cab} to contact terms, some of which contribute to δ^{abc} . Such reduction of terms has been implemented for $\mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)}$, for instance in Ref. [8], and in the relativistic effective Lagrangian as well [3]. Some terms in $\mathcal{L}^{(3)}$ whose coefficients are determined by reparametrization invariance [8] may seem at first glance to give a q^2 dependence to g_{cab} , but a careful calculation shows that this is not so.

Since χ_- is $\mathcal{O}(p^2)$, and since a factor of the spin operator S_ν^μ is needed, the LO tree contributions to the GTD must come from $\mathcal{L}^{(3)}$. One can further argue that there are no contributions from the even-order Lagrangians, $\mathcal{L}^{(2n)}$. The reason is that an even number of derivatives would require factors in the monomial of the form $v \cdot \nabla$ which, when acting on the baryon field, are in effect replaced by $\nabla^2/2M$; the other possibility would be factors of $v \cdot S_\nu$ that vanish.

In the case of SU(2), the Lagrangian $\mathcal{L}^{(3)}$ has been given by Ecker and Mojžiš [8,9]. There are only two terms in $\mathcal{L}^{(3)}$ that are of interest to us, namely the terms \mathcal{O}_{19} and \mathcal{O}_{20} given in Refs. [8,10]. In the scheme used by Ecker and Mojžiš these are finite counterterms.

instance, terms quadratic in the quark masses such as $\text{Tr}(\bar{B}S_v^\mu\chi_+[\nabla_\mu\chi_-, B])$ and others.

The contribution to δ^{abc} from $\mathcal{L}_{GTD}^{(3)}$ is given by

$$\frac{\delta_{GT}^{abc}}{4MB_0} = 2s_0[iF_{19}f^{abc} + D_{19}d^{abc}] + d^{cde}s^d[iF_{19}f^{bea} + D_{19}d^{abe}] + s^c[\frac{2}{3}D_{19} + b_{20}]\delta^{ab} , \quad (16)$$

where

$$s_0 = \frac{1}{3}(m_u + m_d + m_s) , \quad (17)$$

$$s^a = \delta_{a3}(m_u - m_d) - \frac{1}{\sqrt{3}}\delta_{a8}(2m_s - m_u - m_d). \quad (18)$$

In deriving Eq. (16) from Eq. (15) we used the Ward identity:

$$\partial^\mu A_\mu^a = 2s_0 \frac{\delta\mathcal{L}}{\delta p_a} + \frac{1}{3}s^a \frac{\delta\mathcal{L}}{\delta p^0} + d^{abc}s^b \frac{\delta\mathcal{L}}{\delta p_c} , \quad (19)$$

as well as the following correspondence of operators between the heavy baryon and relativistic theories:

$$\bar{B}_v S_v^\mu \partial_\mu p B_v \leftrightarrow -iM\bar{B}\gamma_5 p B , \quad (20)$$

where B and B_v are the relativistic and heavy baryon fields respectively.

The leading terms in the GTD are therefore of order p^2 . There are several relations among the discrepancies that are exact at LO. One of them is the Dashen-Weinstein relation [7]:

$$m_K^2 \left(\frac{g_A}{g_V}\right)^{NN\pi} \Delta^{NN\pi} = \frac{1}{2} m_\pi^2 \left(3 \left(\frac{g_A}{g_V}\right)^{NAK} \Delta^{NAK} - \left(\frac{g_A}{g_V}\right)^{N\pi K} \Delta^{N\pi K}\right). \quad (21)$$

states is in the range of 1800 to 2000 MeV, and thus, one may expect that they only give corrections at the order of 20 to 30%. The meson dominance model can be implemented using an effective Lagrangian in analogy with Ref. [11]. The coupling of the Π' octet to the pseudoscalar current is obtained from the effective Lagrangian:

$$\mathcal{L}_{\Pi'} = \frac{1}{2} \text{Tr}(\nabla^\mu \Pi' \nabla_\mu \Pi') - \frac{1}{2} M_{\Pi'}^2 \Pi'^2 + i d_{\Pi'} \text{Tr}(\Pi' \chi_-) + \dots, \quad (22)$$

where we display only those terms relevant to our problem. Here the Π' octet responds to chiral rotations in the same way as the baryon octet. The matrix element of the divergence of the axial current is given by

$$\begin{aligned} \langle 0 | \partial^\mu A_\mu^a | \Pi^b \rangle &= -\frac{B}{2} d_{\Pi'} \text{Tr}(\lambda^b \{\lambda^a, \mathcal{M}_q\}) \\ &= -\delta^{ab} d_{\Pi'} m_a^2, \end{aligned} \quad (23)$$

and the Π' -baryon coupling can be expressed through the effective Lagrangian:

$$\mathcal{L}_{\Pi' B} = D' \text{Tr}(\bar{B} \gamma_5 \{\Pi', B\}) + F' \text{Tr}(\bar{B} \gamma_5 [\Pi', B]). \quad (24)$$

From Eqs. (23) and (24) one readily obtains the contribution to δ^{abc} :

$$\delta_{\Pi'}^{abc} = -d_{\Pi'} g_{\Pi' B}^{abc} \frac{m_c^2}{q^2 - M_{\Pi'}^2} \approx d_{\Pi'} g_{\Pi' B}^{abc} \frac{m_c^2}{M_{\Pi'}^2}. \quad (25)$$

Here $g_{\Pi' B}^{abc} = \frac{F'}{\sqrt{6}} \text{Tr}(\lambda^b [\lambda^c, \lambda^a]) + \frac{D'}{\sqrt{6}} \text{Tr}(\lambda^b \{\lambda^c, \lambda^a\})$. The current situation is that the couplings of the Π' are not known, and there is no estimate in the literature that one could judge

we do not perform here a full calculation, we do arrive at some interesting observations about such NLO effects by loops. Let us consider the loop diagram in Fig. 2a. We can show that in HBChPT this loop effect on the GTD is $\mathcal{O}(1/M^2)$, and must therefore be suppressed by two powers relative to the LO contribution. Indeed, in HBChPT the diagram is proportional to the following loop integral:

$$-iT^{\mu\nu} \int \frac{d^d k}{(2\pi)^d} \frac{k_\mu k_\nu}{k^2 - m_d^2} \frac{1 + k \cdot v / (2M_f) + \mathcal{O}(1/M_f^2)}{k \cdot v + k^2 / (2M_f) - \delta m_{fa}} \frac{1 + (k + q) \cdot v / (2M_e) + \mathcal{O}(1/M_e^2)}{(k + q) \cdot v + (k + q)^2 / (2M_e) - \delta m_{eb}}, \quad (26)$$

where $\delta m_{ab} \equiv M_a - M_b$, and $T^{\mu\nu}$ is transverse to the four-velocity v . For spin 1/2 baryons in the loop $T^{\mu\nu} \propto S_v^\mu q \cdot S_v^\nu$. It is also easy to show explicitly that $T^{\mu\nu}$ is transverse if one or both lines in the loop are spin 3/2 baryons. From energy-momentum conservation we have

$$q \cdot v = (M_b - M_a) - \frac{q^2}{2M_b}. \quad (27)$$

Using this and the transversity of $T^{\mu\nu}$, the expansion of Eq. (26) shows no q^2 -dependence at $\mathcal{O}(1)$ and $\mathcal{O}(1/M)$. We conclude that the one-loop diagrams considered here must affect the GTD at $\mathcal{O}(1/M^2)$ ¹, and are thus negligible in the large M limit. Another type of one-loop contribution is not suppressed by $1/M$. These are the diagrams involving the insertion of terms from $\mathcal{L}^{(3)}$ as shown in Fig. 2b, which correct the GTD at NLO. Similarly there are NLO two-loop contributions that are of leading order in $1/M$.

It is interesting to comment here on a one-loop calculation in the framework of a relativistic baryon effective Lagrangian, as used in Refs. [3,13]. It turns out that the relativistic

$$\mathcal{J}^{fed}(q^2, M_a, M_b, m_c) = \frac{1}{(4\pi)^2} \mathcal{C}(M_a, M_b, M_e, M_f) \int_0^1 dx \int_0^{1-x} dy \frac{\mathcal{N}(x, y)}{\mathcal{D}(x, y)} , \quad (29)$$

$$\begin{aligned} 2\mathcal{N}(x, y) = & (x + y - 1)(M_a + M_b)^2 - q^2(x + y) + 2(1 - x)M_a M_e + 2xM_a M_f \\ & + 2(1 - y)M_f M_b + 2yM_b M_e + (M_f - M_e)^2 , \end{aligned} \quad (30)$$

$$\mathcal{D}(x, y) = (1 - x - y)(xM_a^2 + yM_b^2 - M_d^2) - M_f^2 x - M_e^2 y + xyq^2 , \quad (31)$$

$$\mathcal{C}(M_a, M_b, M_e, M_f) = (M_b + M_e)(M_a + M_f)(M_e + M_f) . \quad (32)$$

One can readily check that for SU(2) one obtains the result in Ref. [3].

The interesting thing here is that the contribution to the GTD by the loop is not suppressed by $1/M$. Actually, it is nearly constant for baryon masses ranging from a few hundred MeV to an arbitrarily large mass. This result seems at odds with the one from HBChPT, but the two can be harmonized as follows: in the limit of large M it turns out that in the relativistic calculation there are contributions to the loop integral from momenta that are $\mathcal{O}(M)$. M acts in fact as a regulator scale. In HBChPT on the other hand, one is doing a $1/M$ expansion of the integrand, which implies that one is assuming a cutoff in the loop integrals given by a QCD scale. The relativistic and HBChPT frameworks must each lead to the same physical results; in the present case this implies that in order to lead to the same results for the discrepancies, the coefficients F_{19} and D_{19} in $\mathcal{L}^{(3)}$ must be readjusted when going from one framework to the other. In the real world, $M \sim \Lambda_\chi$ and we may use the relativistic calculation as an estimate of this class of loop contributions to the discrepancy. For the discrepancies of interest herein, these loop contributions are small, between ten to

Of course the calculated loop contribution is not all that there is; the inclusion of decuplet baryons in the loop also gives contributions to the discrepancy. (Ref. [14] discusses some $\Delta(1232)$ effects with only two quark flavors.) Using Rarita-Schwinger propagators and three quark flavors, we have checked that the q^2 -dependent part does show an UV divergence in the relativistic framework. HBChPT also permits two-loop contributions at NLO. Currently a more complete calculation of the discrepancy at NLO is underway. [15]

IV. RESULTS

There are only three discrepancies that can be determined from existing data on baryon-pseudoscalar couplings: $\Delta^{NN\pi}$, Δ^{NAK} , and Δ^{NEK} .

Due to the smallness of the u and d quark masses, $\Delta^{NN\pi}$ is necessarily very small, and its determination requires a very precise knowledge of the $g_{\pi NN}$ coupling (g_A and F_π are already known to enough precision, leaving most of the uncertainty in the determination of $\Delta^{NN\pi}$ to the uncertainty in $g_{\pi NN}$). The most recent determination of $g_{\pi NN}$ from NN , $N\bar{N}$ and πN data is by the Nijmegen group [16]. They analyzed a total of twelve thousand data and arrived at $g_{\pi NN} = 13.05 \pm 0.08$. Similar results are obtained by the VPI group [17]. Since the errors quoted are only statistical, in our fit below we will increase the error by about a factor of two in order to roughly account for systematic uncertainties. There is still some disagreement between determinations of $g_{\pi NN}$ by different groups. Larger values have been obtained, such as $g_{\pi NN} = 13.65 \pm 0.30$ by Rugg and Macleod [18], and a similar

$g_{KN\Lambda} = 13.7 \pm 0.4$ and $g_{KN\Sigma} = 3.9 \pm 0.7$. These values are consistent with an earlier analysis by Martin [22], where only an upper bound for $g_{KN\Sigma}$ is given. Using $F_K = 1.22 F_\pi$ and $\left(\frac{g_A}{g_V}\right)^{N\Lambda K} = -0.718 \pm 0.015$ and $\left(\frac{g_A}{g_V}\right)^{N\Sigma K} = 0.340 \pm 0.017$ [20], Eq. (7) gives,

$$\Delta_{\text{expt}}^{N\Lambda K} = 0.17 \pm 0.03 \quad (38)$$

$$\Delta_{\text{expt}}^{N\Sigma K} = 0.17 \pm 0.14 \quad (39)$$

Disregarding SU(2) breaking, which implies that there is no contribution from the term proportional to b_{20} to these discrepancies, we can use the three measured discrepancies to determine the two LO parameters in HBChPT:

$$M F_{19} = 0.4 \pm 0.1 \text{ GeV}^{-1}, \quad (40)$$

$$M D_{19} = 0.7 \pm 0.2 \text{ GeV}^{-1}, \quad (41)$$

where M is here the common baryon-octet mass in the chiral limit. Both choices for $g_{\pi NN}$, given in Eqs. (36) and (37), lead to values for $M F_{19}$ and $M D_{19}$ that agree within the quoted uncertainties. The LO discrepancies resulting from our fit are:

$$\Delta^{NN\pi} = 0.017; \ 0.018, \quad (42)$$

$$\Delta^{N\Lambda K} = 0.17; \ 0.18, \quad (43)$$

$$\Delta^{N\Sigma K} = 0.17; \ 0.19, \quad (44)$$

where the quoted results correspond respectively to the smaller and larger $g_{\pi NN}$ couplings.

suppressed by two powers in HBChPT (this is so because the corrections to the axial-vector couplings and to the discrepancies are of $\mathcal{O}(p^2)$). On the other hand the former possibility would require that the magnitudes of $g_{KN\Lambda}$ and $g_{KN\Sigma}$ be unrealistically large. In fact, Δ^{NAK} and Δ^{NEK} should be close to unity, implying a serious failure of the low energy expansion. Thus, we conclude that only the smaller values of $\Delta^{NN\pi}$, and thus of $g_{\pi NN}$, are consistent. This shows the importance of the current analysis of the GTD in SU(3).

Finally, the coupling constants required in the meson dominance model resulting from our analysis are as follows:

$$d_{\Pi'} F' = 2.4 \pm 0.5 \text{ GeV} \quad (45)$$

$$d_{\Pi'} D' = 4.5 \pm 0.5 \text{ GeV}. \quad (46)$$

Since here F' and D' are baryon-meson couplings, it is not unreasonable that they should have values similar to those of, say, the pion-nucleon coupling. This would imply that the coupling $d_{\Pi'}$ should be a few hundred MeV. This makes the meson dominance picture quite plausible.

In conclusion, we have shown that the GTD in SU(3) is given at leading order by two tree-level contributions, and that the corrections are suppressed by two powers in HBChPT. Some of the loop corrections were calculated explicitly and found to be small. Our leading order analysis indicates a strong preference for a smaller Goldberger-Treiman discrepancy

ACKNOWLEDGEMENTS

We would like to thank Juerg Gasser for allowing us to use material from an earlier unpublished collaboration and for useful discussions. We also thank G. Höhler and Ulf Meißner for useful comments, and Jan Stern for bringing to our attention the Dashen-Weinstein relation. This work was supported by the National Science Foundation through grant # HRD-9633750 (JLG and MS), and # PHY-9733343 (JLG) and by the Department of Energy through contract DE-AC05-84ER40150 (JLG, RL), and in part by Natural Sciences and Engineering Research Council of Canada (RL), the Fundación Antorchas of Argentina (MS) and by the grant # PMT-PICT0079 of the ANPCYT of Argentina (MS).

REFERENCES

- [1] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110** (1958) 1178.
- [2] C. A. Dominguez, Riv. Nuovo Cim. **8** (1985) 1, and references therein.
- [3] J. Gasser, M. E. Sainio and A. Švarc, Nucl. Phys. **B307** (1988) 779.
- [4] E. Jenkins, A. V. Manohar, Phys.Lett. **B255** (1991) 558.
- [5] U-G. Meissner, in "Themes in Strong Interactions", Proceedings of the 12th HUGS at CEBAF, J. L. Goity Editor, World Scientific (1998) 139, and references therein.
- [6] N. H. Fuchs, H. Sazdjian and J. Stern, Phys. Lett. **B 238** (1990) 380.
- [7] R. Dashen and M. Weinstein Phys. Rev. **188** (1969) 2330.
- [8] G. Ecker and M. Mojžiš, Phys. Lett. **B365** (1996) 312.
- [9] N. Fettes, U.-G. Meissner and S. Steininger, Nucl. Phys. A640, 199 (1998).
- [10] H. W. Fearing, R. Lewis, N. Mobed and S. Scherer, Phys. Rev. D56, (1997) 1783.
- [11] G. Ecker, J. Gasser, A. Pich, and E. de Rafael, Nucl. Phys. **B321** (1989) 311.
- [12] J. A. McGovern and M. C. Birse, MC-TH-98-13 preprint (1998). e-Print Archive: hep-ph/9807384.

- [18] D.V. Bugg and R. Machleidt, Phys. Rev. C **52** (1995) 1203.
- [19] B. Loiseau, T.E.O. Ericson, J. Rahm, J. Blomgren and N. Olsson, π N-Newsletter **13** (1997) 117.
- [20] Particle Data Group (C. Caso *et al.*), Eur. Phys. J. C **3**, (1998) 1.
- [21] R. G. E. Timmermans, T. A. Rijken and J. J. de Swart, Phys. Lett. B **257** (1991) 227.
R.E.G. Timmermans, Th. A. Rijken and J.J. de Swart, Nucl. Phys. A **585** (1995) 143c.
- [22] A. D. Martin, Nucl. Phys. B **179** (1981) 33.

FIGURES

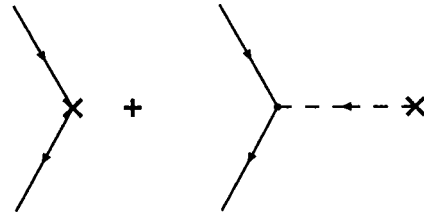


FIG. 1. Diagrams representing the contact term and the pole term in the matrix elements of the divergence of the axial currents. Crosses represent the divergence of axial currents.

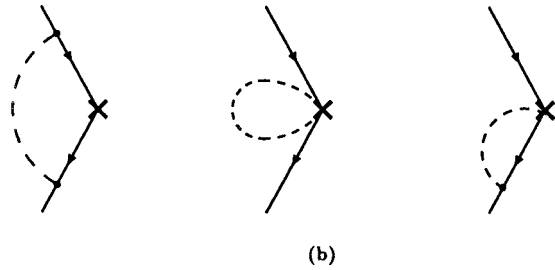
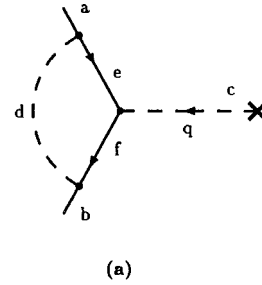


FIG. 2. One loop diagrams that give NLO corrections to the GTD. In (a) the cross represents the divergence of the axial current obtained from the $\mathcal{O}(p^2)$ Lagrangian, and in (b) the same